Physics behind the Hindustani Classical Flute

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Abstract

The Bansuri is a wind instrument that produces sound due to the vibration of air particles in its air column. This project deals with the physics behind the bansuri and its construction. Most phenomena related to this wind instrument can be explained by the famous frequency-wavelength equation [fʎ=v, where f is

frequency, ʎ (lambda) is wavelength, and v is the velocity of sound]. The explanation of subjective quantities of sound (like pitch and loudness), has been done based on principles of physics. Many facts stated in this project have been observed through experimentation rather than the acceptance of results from other sources. The reasons behind occurrences are almost always intuitive but require reasoning. This project aims to explain these occurrences intuitively for a better understanding of the flute and sound in general.

Introduction

The Hindustani classical flute, also called the Bansuri, is a wind instrument, predominantly played in Northern India and Nepal. It comprises of six or seven holes that make up the musical octave. An octave is a definite set of frequencies. These frequencies are in simple mathematical whole number ratios. These frequencies depend on the first note of the octave. The last note of an octave is essentially a repetition of the first. Mathematically, a note which is an octave higher than its corresponding note in the preceding octave, is double the frequency of the predecessor. This *simple ratio* of 2:1, makes these notes pleasing when rendered in a similar manner. In general, *when there is a simple mathematical ratio between the frequencies of two notes, the notes sound pleasing, and are considered musical. When there is no simple mathematical ratio between two notes, the notes are considered noisy and unpleasant.* Keeping this in mind, the musical octave, consisting of seven notes and a repeating note, is designed such that there is always a simple mathematical ratio between the frequency of each of the notes.

Objectives

1) To replicate a glass flute based on bamboo flute
2) To understand the science behind the flute
3) To design a glass flute for any Key.

The Hypothesis

"Frequency is dependent on the inner volume of the flute".

It was assumed that *all bansuris of the same key, should have the same inside volume*. This would mean that two bansuris of different lengths and different internal radii but having the same inner volume should have the same frequency (key). A bamboo flute with D# key was taken as reference. The inner volume of the reference was calculated. A readily available glass tube with appropriate dimensions was taken. The inside volume of the tube was marked out to match the inside volume of the reference. The ratio of length of each of the holes from the closed end to the entire inner length of the reference was calculated. Then with these ratios, the estimated position of the holes was predicted Based on calculations enclosed. With the predicted hole positions, a glass flute was developed.

The table shows the positions of the holes in the reference, the ratios calculated, and the corresponding positions in the designed model. The reference had a length of 370mm and inner diameter of 16mm. The model was designed with an inner diameter of 15.69mm (Based on availability of tube size) and calculated the length to be approximately 385mm.

Name of Hole	Distance of holes from closed end in Reference Bansuri (mm)	Ratio of length of hole from closed end to total length of reference	Corresponding Position of holes in designed model (mm)
Mouth/Blow	2	0.01	2.1
Ga	153.7	0.42	159.8
Re	179.9	0.49	187.1
Sa	206.1	0.56	214.3
Ni	239.9	0.65	249.5
Dha	254.9	0.70	265.1
Pa	288.6	0.79	300.1
Ma’	333.6	0.91	346.9
Volume (cc)	743.93		743.93

Observations :

- All the notes were in tune while ascending and descending.
- Higher octave notes were also in sync with middle octave.
- The key of the flute was flatter compared to the reference by approximately ***14 Hz.***

The following sheet shows the working behind the calculations. Look at the tab called “D#” in the excel spreadsheet, “Flute project Working 1”.

Date of experiment 07-Nov-20
Venue Vertis

Flute Key	**Inner Diameter (mm)**	**Outer Diameter (mm)**	***Outer L* Total (mm)**	***ID L* Open End to Seal**	**Blow Hole from seal**	**Hole 1**	**Hole 2**	**Hole 3**	**Hole 4**	**Hole 5**	**Hole 6**	**Hole 7**	**Volume**
D# middle	16	19	420	370	2	153.7	179.9	206.1	239.9	254.9	288.6	333.6	74393
				Hole Size	10.5	10	9.5	9	9	10	9	9.5	
				Ratios	0.01	0.42	0.49	0.56	0.65	0.69	0.78	0.90	
Tube to be taken for D#	15.69	18.000		385	2.1	159.8	187.1	214.3	249.5	265.1	300.1	346.9	74393

Refer to “Flute project Working 1” for detailed calculations.

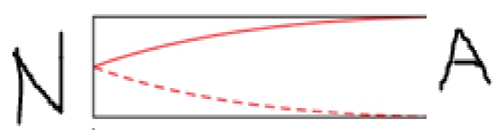
Based on the observations, it was concluded that the Hypothesis was wrong.

What Determines the Key of a Bansuri

After the hypothesis was proven wrong, an attempt was made to understand sound characteristics, wave motion, vibrations in closed end air columns. Following are the salient points in the study:

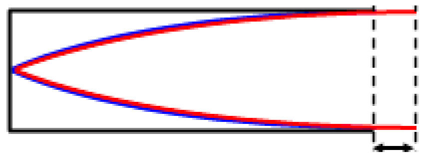
- Sound is a mechanical, longitudinal pressure wave that consists of *compressions and rarefactions*.

- If a graph of pressure versus time of a sound wave is plotted, c*rests, troughs and points of least amplitude* are obtained.
- The crests signify the compressions of the wave, the troughs signify the rarefactions, and the points of least amplitude are symbolic of *nodes*.
- Nodes can be defined as regions in the propagation of a sound wave at which *amplitude is zero*. In other words, node is a region where there is no sound.
- In contrast, *antinodes* are the points where *amplitude is maximum*, i.e., the crests and troughs.
- The Bansuri falls under the category of Closed End Air Column.
- In a bansuri, the node is at the closed or sealed end, as there is limited space for the movement of air molecules. On the other hand, the antinode occurs at the open end of the flue, as the molecules are free move, without obstruction, as shown in the figure:



'N' represents a node, while the 'A' represents an antinode.

- But according to Lord Rayleigh the antinode of a closed air column (like in a flute or bansuri) is just outside the open end of the air column. This is because the air molecules are completely free to move in this region and hence facilitates the formation of an antinode. Therefore, the effective length of the flute changes. The point where the antinode is formed is the effective length of the flute. This is shown in the figure. The arrow represents the length of end correction:



Impact of Length on Frequency

When all holes are closed (including the blowing hole) and air is blown from the open end of the bansuri, the fundamental frequency is obtained. This sound (displacement) is the product of the superposition of 2 waves, originating sound in the negative X axis and the reflected sound in the positive X axis. This is given by

$y = y_1 + y_2 = a \sin 2\pi/\Lambda\,(vt+l) + a \sin 2\pi/\Lambda(vt-l) = 2a \cos(2\pi vt/\Lambda) \sin(2\pi l/\Lambda)$

where a=amplitude, v=velocity of sound in air, t=time and l=length of air column.

Now, the displacement will be maximum when **$\sin(2\pi l/\Lambda)$** is maximum. This happens when **$\sin(2\pi l/\Lambda)$** = +/-1. Therefore,

$2\pi/\Lambda = (2m-1)\,\pi/2$, or $\Lambda = 4l / (2m-1)$

The frequency of a bansuri is dependent on its length. This is because frequency of sound is directly proportional to the velocity of sound in the medium (air in the case of a bansuri) and is inversely proportional to the wavelength of the sound wave. As velocity of sound in a medium cannot be altered, the length of the air column (which is directly proportional to the wavelength) must be altered to change the frequency.

$\Lambda = 4l / (2m-1)$ where 'l' is the length of the air column, and 'm' is a natural number (i.e. m= 1,2,3...). 'm' represents the number of nodes.

$f = v/\Lambda = (2m-1)v/4l$

Therefore, to increase the frequency, the length of the air column must be decreased and vice versa. This is the fundamental principle that governs the sounds produced by the bansuri. As the length of the flute cannot be altered, there are holes which can be covered to change the length of the air column. This is how we can easily change frequency to achieve the octave.

Impact of Area of cross section on Frequency

Even though the area of cross section of the flute (which is dependent on the radius =>$A = \pi r^2$) does not directly affect its frequency, it does matter. This is where end correction of the flute comes into play. The end correction of the flute is directly proportional to the radius of the area of cross section of the flute:

$\mathbf{e=0.6r}$, where ‘e’ is the end correction, and ‘r’ the radius.

Therefore, the effective length of the flute becomes:

$\mathbf{L_e = l_a + e}$, where ‘L_e’ is the effective length of the flute, ‘l_a’ the actual length, and ‘e’, the end correction.

The expression for frequency then becomes:

$$\mathbf{f = (2m-1)\, v / 4\, (l + e) = (2m-1)\, v / 4\, (l + 0.6r)}$$

Impact of Area of playing holes on Frequency

The size of the playing holes has an impact on the frequency. If the size of the holes is increased, then the frequency also increases slightly. This is because, as one increases the size of the holes, the length of the air column decreases. This causes the wavelength to decrease, and hence the frequency increases.

Making the holes smaller has the opposite effect.

Other Important Characteristics of Sound Related to the Bansuri

Amplitude

We have already seen that the radius of cross section of the bansuri affects the frequency only to a small extent. But the radius occupies an important role in the amplitude of sound produced. Greater the radius, greater is the amplitude. This factor helps in making the lower notes, i.e. of lower frequency, more audible. Usually, with a thinner radius, the lower notes are difficult to play and be heard. But with a greater size of the radius, these notes can be played and heard relatively easily. Therefore, the Bass flutes (those which have a lower key or frequency) have a greater radius size.

But with increase in radius size, playing higher notes becomes difficult. Therefore, a compromise must be reached according to the key of the bansuri.

Fundamental Frequency

The fundamental frequency of an instrument is defined as the lowest possible frequency it can play. For a flute or bansuri, the lowest frequency is obtained when all finger holes are closed, and air is blown *softly*. This can be explained by the fact that the length of the air column is maximum and therefore, the wavelength is maximum. This makes the frequency minimum. This can also be understood by the expression for frequency derived earlier. The lowest value of frequency will occur when 'm' is lowest, i.e. m=1:

$$f = (2m-1)\, v / 4\, (l + 0.6r) = v / 4\, (l + 0.6r)$$

Key of the Bansuri

The key of the bansuri is the note that plays "Shadj" or "Sa" of the musical octave. This is achieved by covering the first three holes from the blowing hole, as shown in the figure:

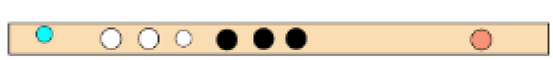


FIGURE SHOWING BANSURI PLAYING SA

Ratio of ID to total length

The ideal ratio of the Inner Diameter to total length should be 1:23 or 4.35%. In designing the D# middle flute, there was no problem in approximately maintaining the ratio, however there were problems while designing the F Bass flute as discussed later.

Tuning of the Bansuri

The bansuri is tuned such that the note that corresponds to A in western music, is 440Hz. This is the reference standard in the Bansuri. The two later flutes designed in this project are tuned such that their corresponding A note is 440Hz.

Redesigning the D# Bansuri in Glass

After understanding the working of the flute as above, a 2nd visit was planned on 13 Nov 2020 to the factory to implement the D# flute.

All flutes with the same Key will have the same effective length. The D# was now designed to have an effective length equal to that of the reference, as enclosed under

Date of experiment 07-Nov-20
Venue Vertis

Flute Key	Inner Diameter	Outer Diameter	*Outer L* Total(cm)	*ID L* OpenEnd to Seal	Blow Hole from seal	Hole 1	Hole 2	Hole 3	Hole 4	Hole 5	Hole 6	Hole 7
D# middle	16	19	420	370	2	153.7	179.9	206.1	239.9	254.9	288.6	333.6
				Hole Size	10.5	10	9.5	9	9	10	9	9.5
			Effective L	374.8								
				Ratios	0.01	0.41	0.48	0.55	0.64	0.68	0.77	0.89
End Correction	4.707											
Tube to be taken for D#	15.69	18.000		370.1	2.0	151.8	177.6	203.5	236.9	251.7	285.0	329.4

Refer to tab “D# redesigned” in the excel spreadsheet, “Flute project Working 1”.

Conclusion: The model had correct frequency for its key. The remaining notes in the octave were also in tune.

Frequency of notes in an Octave for D# (middle)

The notes in the octave have specific frequencies that bear simple whole number ratios. The table below gives the frequencies of the corresponding notes in the octave. Note that the key in this case is D# in the 5th octave (D#$_5$).

Note in Western Nomenclature	Note in Indian Octave	Frequency in Hertz (Hz)
D#$_5$	Sa	622.25
F	Re	698.46
G	Ga	783.99
A	Ma’	880.00
A#	Pa	932.33
C	Dha	1046.50
D	Ni	1174.66
D#$_6$	Sa’	1244.5

Designing a Glass Flute without a Reference

Using the existing ratios for hole positions, an attempt was made to design F Bass without a reference flute of F Bass Key. With this, the flute was designed with a higher inner diameter glass tube of 22 mm. Effective length was calculated using the fundamental frequency and was found to be 668.2 mm. Please find attached the calculation sheet under “F Bass”.

Flute Key	Inner Diameter	Outer Diameter	*Outer L* Total(cm)	*ID L* OpenEnd to Seal	Blow Hole from seal	Hole 1	Hole 2	Hole 3	Hole 4	Hole 5	Hole 6	Hole 7
Fundamental F	246.94		Effective L	668.2								
Wavelength	1.34											
Effective L	0.668			Ratios	0.01	0.41	0.48	0.55	0.64	0.68	0.77	0.89
End Correction	6.6											
Tube to be taken for D#	22	26.000	Actual L	662	6.7	274.0	320.7	367.5	427.6	454.4	514.5	594.7

Refer to tab “F Bass in the excel spreadsheet “Flute project Working 1”.

Conclusions:

- The key and notes of the octave were in tune.

For designing a bansuri without any reference please look at the Java program given in the next section.

Note that this program is only for bansuris having a middle key.

The picture below shows the three bansuris made based on the designed models. The topmost has key D# but is a little flat. The middle bansuri has the key D# and is in tune. The bottom most bansuri is that of F Bass.

Designing a Bansuri using Java

With the Java program given below, any bansuri ***with a middle key*** can be designed. It requires a user input for the key of the flute to be designed and the radius of the flute to be designed. If you want to enter a key that is an accidental (a sharp key or a flat key, ie., not a natural note. Eg. Csharp), capitalize the key, write it as a *sharp (and not a flat),* and do not leave a space between the key and the "sharp". Do not capitalize the "sharp".

For example, if you want to enter F Sharp, write it as *"Fsharp".*

Note: There are no keys/notes called "Bsharp" or "Esharp".

All dimensions regarding hole positions are given (in metres).

```
import java.util.Scanner;
public class BansuriDesign {
   public static void main(String [] args) {
      Scanner console= new Scanner(System.in);
      System.out.print("Enter key in the flute");
      String s= console.next();
      double f=0.0; // Lowest frequency
      double A=440.0, Asharp=466.16, B=493.88, C=523.25, Csharp=554.37,
D=587.33, Dsharp=622.25, E=659.25, F=698.46, Fsharp=739.99, G=783.99,
Gsharp=830.61;
      switch(s) {
         case "A": f=Dsharp/2; break;
         case "Asharp": f=E/2; break;
         case "B": f=F/2; break;
         case "C": f=Fsharp/2; break;
         case "Csharp": f=G/2; break;
         case "D": f=Gsharp/2; break;
```

```
        case "Dsharp": f=A/2;break;
        case "E": f=Asharp/2; break;
        case "F": f=B/2; break;
        case "Fsharp": f=C/2; break;
        case "G": f=Csharp/2; break;
        case "Gsharp": f=D/2;
      }
      System.out.print("Enter inner radius of tube in metres as a decimal");
      double r=console.nextDouble();
      double v=330; // velocity of sound in air
      double le=0; // Effective length
      double la=0; // Actual length
      double wl=0; // wavelength
      double bh=0, h1=0, h2=0, h3=0, h4=0, h5=0, h6=0, h7=0;

      wl=v/f;
      le=wl/4;
      la=le-(0.6*r);
      System.out.println("Total actual length of flute is "+la+"m");
      bh = 0.01*le;
      System.out.println("Length to blow hole from closed end="+bh+"m");
      h1 = 0.41*le;
      System.out.println("Length to middle of 1st hole from closed end="+h1+"m");
      h2 = 0.48*le;
      System.out.println("Length to middle of 2nd hole from closed
end="+h2+"m");
      h3 = 0.55*le;
      System.out.println("Length to middle of 3rd hole from closed
end="+h3+"m");
      h4 = 0.64*le;
      System.out.println("Length to middle of 4th hole from closed
end="+h4+"m");
      h5 = 0.68*le;
```

```
        System.out.println("Length to middle of 5th hole from closed
end="+h5+"m");
        h6 = 0.77*le;
        System.out.println("Length to middle of 6th hole from closed
end="+h6+"m");
        h7 = 0.89*le;
        System.out.println("Length to middle of 7th hole from closed
end="+h7+"m");
    }
}
```

Conclusion

These were the conclusions drawn from the Hypothesis:

- The ratios of the hole positions seemed to be correct.
- Volume is ***not*** a determinant for the frequency of a flute, and neither is the inner radius and hence area of cross section.
- Only the length of the flute mattered. Since the length of the model was more than the reference, it had a lower key frequency.
- Hypothesis was proven wrong.

The bansuri is a seemingly simple instrument with simple physics behind its working. Decoding this physics brought out some unexpected results.

Scope for further study

- Designing Bass flutes with greater inner diameter for easier blowing.
- A redesigning of the ratios to enable closer finger holes.

Bibliography

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5) Experiments, flute design and implementation courtesy - Vertis (glass factory)